Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Discrete Structures/Дискретные структуры
major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 1 (fall) - Exam

Academic hours: 60 AH in total, including:
- lectures: 30 AH.
- seminars: 30 AH.
- laboratory practical: 0 AH.

Independent work: 45 AH.

Exam preparation: 30 AH.

In total: 135 AH, credits in total: 3

Author of the program: A.B. Daynyak, candidate of physics and mathematical sciences, associate professor, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

Annotation

This is a basic course in discrete mathematics and combinatorics. He wants mathematical intuition to be useful to him. We studied standard things from combinatorics, graph theory, asymptotic analysis.

1. Study objective

Purpose of the course

studying the mathematical foundations of modern combinatorics, as well as preparing students for further independent work in the field of combinatorial problems of applied mathematics, physics and economics.

Tasks of the course

study of the mathematical foundations of modern combinatorics;

acquisition by students of theoretical knowledge in the field of combinatorial analysis of problems arising in practice;

mastering the analytical and algebraic apparatus of discrete mathematics and gaining skills in working with basic discrete structures.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-6 Determine priorities and ways to improve performance through self-assessment	UC-6.1 Achieve personal growth and professional development, determine priorities and ways to improve performance
	UC-6.2 Evaluate performance results in correlation with the set objectives and applied methods
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.1 Demonstrate expert knowledge of research basics in the field of ICTs, philosophy and methodology of science, scientific research methods, and apply skills to use them

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

fundamentals of combinatorics and asymptotic combinatorial analysis;
fundamentals of the theory of generating functions and applications of the theory to enumeration problems of combinatorics;
the foundations of the Mobius theory of inversion and its application to enumerative problems of combinatorics;
fundamentals of graph theory: planarity, isomorphism, Eulerianity, Hamiltonianity, chromatic number, chromatic polynomial and Tatt polynomial, trees, multigraphs, digraphs, tournaments, admissible sequences of degrees of vertices, number of connected graphs with a given number of vertices and edges (Cayley formula for the number of trees and its generalization);
fundamentals of the theory of hypergraphs: the theorems of Erdшas-Ko-Rado, Frankl-Wilson and Alsvede-Khachatryan, intersection graphs and edge graphs, chromatic numbers of Kneserov graphs;
fundamentals of the theory of random graphs: connectivity, distribution of tree components, evolution of a giant component, the concept of a random web graph;
fundamentals of combinatorial geometry and its connection with the theory of graphs and hypergraphs;
the fundamentals of coding theory and its connection with the theory of graphs and hypergraphs: Hadamard matrices, error correction codes, Hamming and Reed - Muller codes;
main probabilistic methods in combinatorics: linearity of mathematical expectation, alternation method, second moment method and estimation of large deviations;
basic linear algebraic methods in combinatorics: linear independence of polynomials over a finite field;
basic topological methods in combinatorics: application of the Borsuk – Ulam – Lyusternik – Shnirelman theorem;
the basics of Ramsey theory: Ramsey numbers for graphs and hypergraphs, bipartite Ramsey numbers, constructive estimates;
the fundamentals of the theory of representative systems for graphs and hypergraphs, including the concept of the Vapnik – Chervonenkis dimension and its application to problems of combinatorial geometry and mathematical statistics;
fundamentals of extreme combinatorics: Turan's theorem and its refinement for distance graphs.

be able to:

calculate the number of different combinatorial objects: combinations, placements, permutations, cyclic sequences;
prove combinatorial identities;
calculate approximate values (asymptotics) of combinatorial expressions;
make and solve recurrence relations;
prove the various properties of graphs and hypegraphs;
solve extreme combinatorics problems;
build representative systems for graphs and hypergraphs;
solve Ramsey problems;
evaluate chromatic numbers of graphs, construct Tatt polynomials and chromatic polynomials;
Build error correction codes.

master:

independent work skills;
skills of mastering a large amount of information;
culture of formulation and modeling of combinatorial problems;
probabilistic method in combinatorics;
linear-algebraic method in combinatorics;
topological method in combinatorics;
method of generating functions;
using the Mobius method.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

		Types of training sessions, including independent work

№	Topic (section) of the course	Lectures	Seminars	Laboratory practical	Independent work
1	Recall of basic objects and facts	4	4		6
2	Proof techniques	4	4		6
3	Elementary counting	4	4		6
4	Counting	4	4		6
5	Ramsey theory	4	4		6
6	Introduction to probabilistic method	4	4		6
7	Introduction to linear algebraic methods	6	6		9
AH in total		30	30		45
Exam preparation		30 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 1 (Fall)

1. Recall of basic objects and facts

Sums, sets, sequences, floor and ceiling functions, basic notions of graph theory.

2. Proof techniques

Induction vs. minimal counterexample, potential method, pigeonhole principle vs. taking means, double counting.

3. Elementary counting

Multiplication principle, inclusion-exclusion

4. Counting

Recurrence relations (isolating an element), Redfield—Polya theory (employing symmetries).

5. Ramsey theory

Ramsey numbers, Ramsey theorem for finite graphs and hypergraphs,obtaining bounds for Ramsey numbers.

6. Introduction to probabilistic method

Existence proofs via counting, applications of Markov inequality, Lovasz' local lemma.

7. Introduction to linear algebraic methods

Rank bound (Fisher's inequality), quadratic forms (Graham—Pollack theorem), introduction to linear error-correcting codes.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Вероятностный метод [Текст] : учеб. пособие для вузов / Н. Алон, Дж. Спенсер ; пер. 2-го англ. изд. под ред. А. А. Сапоженко .— М. : БИНОМ. Лаб. знаний, 2007, 2013 .— 320 с.

2. Дискретная математика [Текст] / А. Н. Макоха, П. А Сахнюк, Н. И. Червяков - М.Физматлит,2005

3. Дискретная математика: задачи и решения [Текст] / Г. И. Просветов - М.БИНОМ. Лаб. знаний,2008, 2011

Additional literature

1. Сборник задач по дискретному анализу. Комбинаторика. Элементы алгебры логики. Теория графов [Текст] : учеб. пособие для вузов / Ю. И. Журавлев [и др.] ; М-во образования Рос. Федерации, Моск. физ.-техн. ин-т (гос. ун-т) .— 2-е изд. — М. : МФТИ, 2000, 2004 .— 100 с.

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

9. Guidelines for students to master the course

1. It is recommended to successfully pass the test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

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Author: A.B. Daynyak, candidate of physics and mathematical sciences, associate professor, associate professor

1. Competencies formed during the process of studying the course

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Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.1 Demonstrate expert knowledge of research basics in the field of ICTs, philosophy and methodology of science, scientific research methods, and apply skills to use them

2. Competency assessment indicators

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the foundations of the Mobius theory of inversion and its application to enumerative problems of combinatorics;

fundamentals of graph theory: planarity, isomorphism, Eulerianity, Hamiltonianity, chromatic number, chromatic polynomial and Tatt polynomial, trees, multigraphs, digraphs, tournaments, admissible sequences of degrees of vertices, number of connected graphs with a given number of vertices and edges (Cayley formula for the number of trees and its generalization);

fundamentals of the theory of hypergraphs: the theorems of Erdшas-Ko-Rado, Frankl-Wilson and Alsvede-Khachatryan, intersection graphs and edge graphs, chromatic numbers of Kneserov graphs;

fundamentals of the theory of random graphs: connectivity, distribution of tree components, evolution of a giant component, the concept of a random web graph;

fundamentals of combinatorial geometry and its connection with the theory of graphs and hypergraphs;

the fundamentals of coding theory and its connection with the theory of graphs and hypergraphs: Hadamard matrices, error correction codes, Hamming and Reed - Muller codes;

main probabilistic methods in combinatorics: linearity of mathematical expectation, alternation method, second moment method and estimation of large deviations;

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be able to:

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3. List of typical control tasks used to evaluate knowledge and skills

Current control

[The questions below are written in LaTeX notation.]

What is a polynomial coefficient?

How many 3 repetitions can you choose from the ten-element set?

Arrange in ascending order the numbers $A_7^2, \, \ol{A}_7^2, \, C_7^2, \, \ol{C}_7^2$.

The set A has five elements, and the set B has seven elements. What is $|A \cup B|$ if $|A \cap B| = 2$?

Let A be the set of all even numbers not exceeding one hundred. Find $\indicator{A}(5), \, \indicator{A}(76), \, \indicator{A}(510)$?

Let the object x belong to exactly five of the sets $A_1, \, A_2, \, \ldots, \, A_{10}$. What is the product $\prod_{k=1}^7 \indicator{A}(x)$?

Remembering the symmetry and unimodality properties of binomial coefficients, as well as the definitions of combinatorial numbers, arrange the numbers in ascending order: $\binom{1000}{450}$, $\binom{1000}{670}$, $\ol{A}_{1000}^{450}$, $\ol{C}_{1000}^{450}$.

Formulate trans-inequality. How can you prove it using the potential method?

Can you think of an example of a natural n for which there is no multiple of it like $333 \ldots 300 \ldots 00$?

How many edges are there in K_n? What is the eccentricity of the vertices of this graph? What is its diameter? What is his click number and independence number?

What alternative ways can you define a class of trees, other than "connected graphs without cycles"?

How many edges are in the tree for 2013 vertices?

Can a tree with 2013 vertices contain an independent set with 2012 vertices?

Can a complete bipartite graph be a tree?

What are the chromatic number and chromatic index of the chain at 11 vertices? The same question for a cycle on 11 peaks. What is the chromatic number of an arbitrary tree?

In a graph whose degree of all vertices does not exceed 3, there is a cycle of length 2013. What is the chromatic index of this graph?

What is the number of distinct trees on the vertex set $\{1,2, \ldots, 7\}$ with a power sequence $(1,1,1,2,2,2,3)$?

There are no independent sets of size 11 in a graph with 150 vertices. Why can it be argued that such a graph cannot be correctly colored with less than fifteen colors?

Give an example of packing a planar graph for which Euler's formula \emph{does not hold}.

Are there graphs that can be laid on a plane, but not on a sphere?

Is it possible to fit a graph with 100 vertices and 300 edges on a plane?

Give an example of a planar graph whose chromatic number is $ 4 $. Is it true that any planar graph has a chromatic number of at most four?

What degree of vertices can be guaranteed in a planar graph?

Formulate Hall's theorem.

Where in the proof of the correctness of Fleury's algorithm is the lemma on the absence of bridges in connected graphs with even vertex degrees used?

Give an example of a graph that has a Hamiltonian cycle but does not satisfy Ore's conditions.

Does a $ 50 $ -regular graph necessarily have a Hamiltonian cycle on $ 100 $ vertices?

How many vertices are in the De Bruyne graph, which is used to construct a universal De Bruyne sequence of order $ 2013 $?

What is the structure of a graph on $ 10 $ vertices with the maximum number of edges, in which there are no $ 4 $ cliques?

There are no $ 5 $ clicks in the graph with $ 10 $ vertices. What is the maximum number of edges such a graph can have?

How can we obtain from Turan's theorem an estimate for the \ emph {minimal} number of edges in a graph without \ emph {independent sets} of a given size?

What is a correct coloring of a hypergraph?

What is the vertex cover of a hypergraph, what else is it called? What is Matrix Depth?

Give an upper bound for the cardinality of the greedy coverage of the matrix (with a lower bound on the number of ones in a column). Reformulate this theorem in terms of hypergraph coverings.

The lecture builds a matrix on which the greedy algorithm works "very suboptimally". How many units are there in this matrix (for every $ a $)?

What is a permanent?

Do different or the same cyclic words match $ aabba $ and $ baaab $?

How to calculate the number of unordered partitions of $ 50 $?

How many ordered partitions of $ 5 $?

Build a partition diagram $ 13 = 5 + 3 + 2 + 2 + 1 $. Build a diagram that is dual to the previous one. What partition does it correspond to?

Let $ p _ {\ text {different}} (N) $ denote the number of unordered partitions of the number $ N $ into different terms. Why does Euler's partition theorem imply that $ p _ {\ text {distinct}} (1001) $ is even?

What automorphisms does the graph $ K_n $ have, the chain on $ n $ vertices, and the graph $ K_ {n, n} $?

Does the set of all odd numbers form a group with respect to addition? And a lot of even numbers?

This question uses multiplicative notation. Let $ G $ be a group and $ a, b \ in G $. Make sure $ (ab) ^ {- 1} = b ^ {- 1} a ^ {- 1} $.

What is the composition of permutations $ 1 \ to 1, \, 2 \ to2, \, 3 \ to4, \, 4 \ to5, \, 5 \ to3 $ and $ 1 \ to5, \, 2 \ to4, \, 3 \ to1, \, 4 \ to3, \, 5 \ to2 $? Write down the inverse permutations of the data.

How many elements are there in the $ \ bbZ_ {12} $ and $ \ bbZ_ {12} ^ \ times $ groups?

What is element order?

What is a Cyclic Group?

Find $ \ phi (15) $.

Why is the remainder of $ 5 ^ {16} $ divided by $ 17

4. Evaluation criteria

Checklist for the exam:

1. Estimates for factorials and binomial coefficients. Estimates for C_n ^ {n / 2} using identity. Stirling formula (b / d). The notation (c + o (1)) ^ n. Estimates of binomial coefficients of the form C_n ^ {[an]}, a \ in (0,1). Similar results for polynomial coefficients. Asymptotics for C_n ^ k for k ^ 2 = o (n). Estimates of the same value for large k. Asymptotics for C_n ^ {n / 2} / C_n ^ {n / 2-x}.

2. Definition of a graph, digraph, multigraph, pseudograph, etc. Equivalent wood definitions (4 pieces). Cayley's formula. Unicyclic graphs. The exact formula for the number of different unicyclic graphs (one must be able to prove the lemma on the number of forests) and asymptotics. A review of the results on the number of connected graphs with a given number of vertices and edges.

3. Routes in graphs (chains, loops). Euler graphs (3 equivalent definitions). A criterion for the Eulerian property of a digraph. Sequences and Counts of de Bruyne. The rule "zero is better than one."

4. Independent sets and clicks. Independence number and click number. The connection between them. Turan's theorem on the number of edges in a graph with a given number of vertices and the number of independence. Corollary with an asymptotic estimate in the case of a sequence of graphs. Remote graphs. Estimation of the number of edges in a distance graph on a plane. The concept of a simplex in space. An estimate of the number of edges of a distance graph in an arbitrary dimension. Corollary with asymptotics. Comparison with the usual Turan theorem.

5. Determination of the planarity of the graph. Image on a sphere. The definition of the face. Euler's formula. Estimates of the number of edges resulting from the Euler formula. The nonplanarity of the graphs K_5 and K_ {3,3} as a consequence of the estimates. Pontryagin-Kuratovsky theorem.

6. Hamiltonian cycles and chains. A sufficient condition for the Hamiltonian graph.

7. Random graphs. Inequalities of Markov and Chebyshev (we must remember d-va). Inequality for random walk. Moments and factorial moments. The formula of treatment (b / d, but as a bonus exercise). Poisson approximation theorem (b / d, but with d-v in a simplified situation). Commentary in terms of the moments of a Poisson random variable. Connectivity of a random graph: cases p = cln n / n for c> 1 and c <1 with proof; the theorem on (ln n + c + o (1)) / n is only a statement. Giant Component Theorem (b / d). The wood component theorem: 5 situations, 3rd - b / d.

8. Chromatic polynomial: definition, calculation for a complete graph, an empty graph, chain, cycle. General formula expressing oil on canvas Count through oil on canvas a graph with an ejected edge and with a contracted edge. Theorem: xm is a polynomial of degree n with integer alternating coefficients, the leading coefficient is 1, and the following coefficient is | E |. Application: oil on canvas any tree. A note on the difficulty of calculating oil on canvas Determination of the spanning number (total number of spanning forests in the graph), recurrence relations with contractions and removals.

9. Definition of the Tatt polynomial. The correctness theorem for the definition of the Tatt polynomial (b / d). The universal property of the Tatt polynomial. Examples of application of the universal property: spanning number, number

subgraphs with the same number of connected components, the number of acyclic

subgraphs, chromatic polynomial (last three as mandatory tasks).

10. Chromatic number, independence number, clique number and relations between them. Comparison of estimates of the chromatic number in terms of clique number and independence number in terms of random graphs: one "almost always" is much better than the other (distribution of clique number and independence number). Bollobash theorems on the chromatic number of a random graph (b / d). Explanations for them: 1) p = o (1 / n ^ 2); 2) p = o (1 / n); 3) p = c / n, c <1, - b / d; 4) the function from the second Bollobash theorem can tend to infinity.

11. Hamiltonian chains in tournaments: the lower score with d-vom, the upper - b / d.

12. Hypergraphs. Erdös – Ko – Rado theorem (maximum number of edges in a 1-intersecting hypergraph). t-intersecting hypergraphs, f (n, k, t). An example where the lower bound f (n, k, t) \ ge C_ {n-t} ^ {k-t} is obviously not accurate. History of successive advances in the problem: Erdös – Ko – Rado theorem (general case), Frankl's theorem, Wilson's theorem, Alsve de Khachatryan's theorem (all b / d, but with detailed comments). Intersection graph for a complete homogeneous hypergraph. Its clique number and independence number.

13. Kneserov graph (disjoint graph for a complete homogeneous hypergraph). An upper estimate of its chromatic number. Simple lower bounds. Examples of concrete Kneser graphs. The Lovas theorem on the chromatic number of a Kneser graph (here the Borsuk – Ulam – Lyusternik – Shnirelman theorem in different formulations, but with proof only in the case of a plane and three-dimensional space).

14. The value of m (n, k, t). Exact value for m (n, 3,1): explicit construction and induction estimate. Linear-algebraic estimate for m (n, 3,1). A similar estimate for m (n, 5,2) and its asymptotic unimprovability. The Frankl-Wilson general theorem for m (n, k, k-p). A note about a difficult "module". An example where k = n / 2, p is the minimal prime with the condition k-2p <0. The "pathos" of the example. The "accuracy" of the example (above and below (1.754 ... + o (1)) ^ n).

15. Transversals (systems of various representatives). Hall theorem. Perm-Nent. Frobenius – Koenig Theorem.

16. Systems of common representatives (s.p.p.). Reformulation in terms of covering matrices. The value of (n, s, k). "Trivial" lower and upper bounds. Upper bound using a greedy algorithm. Constructive lower bound. Probabilistic lower bound. The relationship of the upper and lower bounds.

17. Certificates ("witnesses") of binary vectors. Bondi's theorem. Top grade average certificate size.

18. Theory of Ramsey. Ramsey numbers R (s, t): exact values for s = 1,2; the upper Erdos – Szekeres estimate s + ts, its corollary for diagonal Ramsey numbers; lower bound for diagonal numbers using a simple probabilistic method. Ramsey theorem for hypergraphs (b / d). Ramsey theorem for infinite sets.

19. Lovas lemma: symmetric and general cases. The idea of clarifying the lower bound for Ramsey numbers with its help.

Exam ticket examples

Ticket number 1

1. Certificates ("witnesses") of binary vectors. Bondi's theorem. Top grade average certificate size.
2. Routes in graphs (chains, loops). Euler graphs (3 equivalent definitions). A criterion for the Eulerian property of a digraph. Sequences and Counts of de Bruyne. The rule "zero is better than one."

Ticket number 2

1. Kneserov graph (disjoint graph for a complete homogeneous hypergraph). An upper estimate of its chromatic number. Simple lower bounds. Examples of concrete Kneser graphs. The Lovas theorem on the chromatic number of a Kneser graph (here the Borsuk – Ulam – Lyusternik – Shnirelman theorem in different formulations, but with proof only in the case of a plane and three-dimensional space).
2. Definition of planar graph. Image on a sphere. The definition of the face. Euler's formula. Estimates of the number of edges resulting from the Euler formula. The nonplanarity of the graphs K_5 and K_{3,3} as a consequence of the estimates. Pontryagin-Kuratovsky theorem.

Assessment “excellent (10)” is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment “excellent (9)” is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one’s own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment “excellent (8)” is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (7)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment "satisfactory (3)" is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment "unsatisfactory (2)" is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment "unsatisfactory (1)" is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During examination the student are allowed to use the program of the discipline.